Sl. N **C–JTT–M–TUC**

STATISTICS—III

Time Allowed : Three Hours	Maximum Marks : 200

INSTRUCTIONS

Candidates should attempt FIVE questions in ALL including Question Nos. 1 and 5, which are compulsory. The remaining THREE questions should be answered by choosing at least ONE question each from Section—A and Section—B.

The number of marks carried by each question is indicated against each.

Answers must be written only in ENGLISH.

(Symbols and abbreviations are as usual, unless otherwise indicated.)

If any data/value is to be assumed for answering a question, the same must be mentioned clearly.

All parts and sub-parts of a question being attempted must be completed before moving on to the next question.

Section—A

1. *(a)* Show that for a simple random sample

$$s^2 = \sum_{i=1}^{n} (y_i - \bar{y})^2 / (n-1)$$

is an unbiased estimate of S^2.

(b) Let y_i be the y-value of the ith unit and P_i be corresponding selection probability, $i = 1, 2, \ldots, N$. Then, based on a sample of size n drawn with replacement, show that an unbiased estimator for population total is

$$\hat{Y}_{\text{PPS}} = \frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{P_i}$$

(c) Let the blocks of a BIBD be (1, 2), (1, 3) and (2, 3). Obtain the non-zero eigen-value of C-matrix of BIBD and hence estimates of treatments.

(d) What is a 2^3 factorial experiment? Discuss a method to estimate all main effects and interaction effects considering replication size r. Also write its ANOVA table.

(e) A factorial experiment 2^6 is conducted into a block of size 8 by confounding the interaction effects ABC, $ACDE$ and BCD. Write the total number of blocks per replication, number of generalized confounded interactions. Write all generalized interactions. Is it possible to save 2-factor interaction from being confounded in the experiment? 8×5=40

2. (a) Obtain an unbiased estimator of the gain due to PPSWR sampling as compared to SRSWR.

(b) Show that the ratio estimator is better than the one based on SRSWOR if $\rho > \frac{1}{2}$ when $C_X = C_Y$.

(c) Show that the bias of the usual regression estimator is $-\operatorname{cov}(\bar{x}, b)$.

(d) Show that mean of a systematic sample is more precise than the mean of a simple random sample of size n under a certain condition to be obtained by you.

10×4=40

3. (a) Discuss the construction of a BIBD with parameters $v = 9$, $b = 12$, $r = 4$, $k = 3$ and $\lambda = 1$ using EG(N, S) method.

(b) Let four treatments be arranged in three blocks. The arrangement is given below :

Block 1 : $[t_1 \; t_2 \; t_3 \; t_4]$
Block 2 : $[t_2 \; t_3 \; t_4 \; t_1]$
Block 3 : $[t_3 \; t_4 \; t_1 \; t_2]$

Identify the block design. Write its parameters. If the treatment t_4 in Block 3 is missing, estimate the missing value. Write its ANOVA table.

(c) Discuss symmetrical BIBD. For an SBIBD, under usual notations, show that

$$|N| = r(r - \lambda)^{\frac{v-1}{2}}$$

(d) Discuss the layout of a Latin square design. Give its example. Write its model (one observation per experimental unit). Give its ANOVA table. Write null hypothesis. Give comments on acceptance and rejection of null hypothesis. 10×4=40

4. (a) Discuss the allocation of a sample to different strata in stratified sampling. Explain proportional allocation and hence obtain the variance of mean of a stratified sample under proportional allocation.

(b) Explain error of measurement. Write its model. For the given model $Y_{ij} = x_i + \alpha_j + e_{ij}$, obtain its sample mean and then show that $\bar{Y}_{..}$ is not an unbiased estimator provided $n_{.j} = \frac{n}{m} = \bar{n}$ and $n_{i.} = \frac{n}{h} = p$.

(c) Construct a group divisible design with parameters $v = 8 = b$, $r = 3 = k$, $\lambda_1 = 0$, $\lambda_2 = 1$, $m = 4$, $n = 2$. Further identify the type of GD design.

(d) Estimate the treatment effect of BIB design under intrablock analysis using C-matrix of the block design. 10×4=40

Section—B

5. *(a)* What are the different methods for measurement of trend? Discuss the method of fitting a straight line using least squares method.

   (b) Discuss link relative method to estimate seasonal fluctuations, with appropriate illustrations.

   (c) Consider a model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + (\varepsilon - \bar{\varepsilon})$$

   with $\Sigma X_1 = \Sigma X_2 = 0$, where

$$X_2 = 2\theta X_1 + V, \quad \Sigma X_1^2 = \Sigma X_2^2 = 1,$$

$$\Sigma V = 0, \quad \Sigma X_1 V = 0$$

   Does the model contain multicollinearity? If yes/no, explain. Also obtain $V(\hat{\beta}_i)$, $i = 1, 2$ and cov $(\hat{\beta}_1, \hat{\beta}_2)$.

   (d) Let the demand function be expressed as $P = 4 - 5X^2$. For what value of X, the elasticity of demand will be unitary?

   (e) Let true X be related as $X = X^* + \varepsilon_X$, where ε_X is the measurement error in X. Now for the given regression relation $Y = X^* \beta + \varepsilon_Y$, show that measurement errors in one variable are in the limit uncorrelated with the measurement errors in the other variable and measurement errors in both variables are uncorrelated with true values.

8×5=40

6. *(a)* Write various types of identification problem. Identify only the first equation of the following simultaneous equations :

$$Y_1 = \Gamma_{21} Y_2 + \beta_{11} X_1 + \varepsilon_1$$
$$Y_1 = \Gamma_{22} Y_2 + \beta_{22} X_2 + \beta_{32} X_3 + \varepsilon_2$$

(b) Discuss forecasting accuracy and Theil's U coefficient.

(c) Let $Y = X\beta + \varepsilon$, where $\varepsilon \sim N(0, \sigma^2 \Omega)$. Obtain the estimate of β and its variance, where Ω is known symmetric and positive definite of order n.

(d) Under heteroscedasticity, show that OLS estimator is less efficient than the weightage least squares estimator even though OLS is an unbiased estimator.

10×4=40

7. *(a)* Obtain the general solution of first-order autoregression model.

(b) Obtain the correlogram for the series $U_{t+1} = aU_t + \varepsilon_{t+1}$, $|a| < 1$ with $E(\varepsilon) = 0$, provided $\varepsilon_{t+1} = b\varepsilon_t + \eta_{t+1}$, $|b| < 1$, and successive values of η are independent.

(c) Describe Pareto's law of income distribution and its graph. What type of data is required for determining it?

(d) Using curves of concentration, discuss the formulation of the problem of distribution of income. 10×4=40

8. *(a)* Discuss Engel's law and Engel's curve. Explain Engel's curve for constant price.

(b) If the demand curve is of the form $p = ae^{-kx}$, where p is the price and x is the demand, prove that the elasticity of demand is $\frac{1}{kx}$. Hence deduce the elasticity of demand for $p = 10e^{-\frac{x}{2}}$.

(c) Show that sum of squares due to sample residual is an unbiased estimator.

(d) Discuss the practical consequences of autocorrelation. Show that

$$V(U_t) = \frac{\sigma^2}{1-\rho^2}$$

10×4=40

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